which I had the honour to lay before the Society in December 1860, and which has since been published in the 'Philosophical Transactions.' I commence this paper with some extensions of the method given in the former memoir for resolving functions of non-commutative symbols into binomial factors. I then explain a method, analogous to the process for extracting the square root in ordinary algebra, for resolving such functions into equal factors. I next investigate a process for finding the highest common internal divisor of two functions of non-commutative symbols, or, in other words, of finding if two linear differential equations admit of a common solution. After this, I give a rule for multiplying linear factors of non-commutative symbols, analogous to the ordinary algebraical rule for linear algebraical factors. I then resume the consideration of the binomial theorem explained in the former memoir. Two new forms of this binomial theorem are here given; and the method by which these forms are proved identical will, I hope, be considered an interesting portion of symbolical algebra, and as exhibiting in a remarkable manner its peculiar nature.

II. "On Internal and External Division in the Calculus of Symbols." By William Spottiswoode, Esq., M.A., F.R.S. Received January 8, 1862.

(Abstract).

Continuing my researches in the calculus of symbols, I have been led to investigate the most general case of division, viz. that wherein a function of any degree n in π is divided, (1) internally, (2) externally, by another function of any other degree m in π. The investigations here subjoined give (1) the various terms of the quotient, together with their laws of derivation both by actual division and otherwise; (2) the final remainder, and thence the conditions that the divisor may be a factor, internal or external as the case may be, of the dividend. An example has been added in each case by way of illustrating the processes. A remarkable reciprocal relation subsisting between the functions (Φ), of the coefficients (ϕ) of the dividend, and the corresponding functions (Ψ) of the coefficients (ψ) of the divisor is exhibited, at the end of the paper.

I have confined myself throughout to that branch of the calculus wherein the functions treated of are arranged according to powers of π; that wherein they are arranged according to powers of ϱ has been already more fully discussed by Mr. Russell.

III. "On the Absorption and Radiation of Heat by Gaseous Matter."—Second Memoir. By JOHN TYNDALL, Esq., F.R.S. Received January 9, 1862.

(Abstract.)

Resuming with a new apparatus his experiments on the influence of chemical combination on the absorption and radiation of heat by gases, the author in the present investigation first examines the deportment of chlorine as compared with hydrochloric acid, and of bromine as compared with hydrobromic acid, and finds that the act of combination, which in each of these two cases notably diminishes the density of the gas, and renders the coloured gas perfectly transparent to light, renders it more opake for obscure heat. He also draws attention to the fact that sulphur, which is partially opake to light, is transparent to 54 per cent. of the rays issuing from a source of 100° C., while its compound, heavy spar, which is sensibly transparent to light, is quite opake to the rays from a source of 100° C. He demonstrates, in confirmation of Melloni, the transparency of lampblack in thin layers, and shows how irreconcileable its deportment to radiant heat is with the idea generally prevalent at the present day, that lampblack absorbs heat of all kinds with the same intensity.

He has repeated all his experiments with gases, using a different source of heat, and finds the result still more pronounced than formerly, that the compound gases far transcend the elementary ones in absorptive power. Taking air as unity, ammonia at 30 inches tension is 1195,—this latter figure representing *all the heat* that issued from the source. A layer of ammonia 3 feet long is *perfectly black* to heat emanating from an obscure source. The coloured gases chlorine and bromine, though much superior in absorptive power to the transparent elementary gases, are exceeded in this respect by every compound gas that has been hitherto examined. When instead